Black Hole Formation in Bidimensional Dilaton Gravity Coupled to Scalar Matter Systems[1]

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Abstract

This work deals with the formation of black hole in bidimensional dilaton gravity coupled to scalar matter fields. We investigate two scalar matter systems, one described by a sixth power potential and the other defined with two scalar fields containing up to the fourth power in the fields. The topological solutions that appear in these cases allow the formation of black holes in the corresponding dilaton gravity models.

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1 Introduction

The coupling of scalar matter fields with bidimensional dilaton gravity, originally proposed by Callan, Giddings, Harvey and Strominger (CGHS) [1] has attracted attention due to connections with black hole physics [2], specially within the context of formation and evaporation of black holes. In cosmology the dilaton field appears to be important, and so several mechanisms for cosmological dilaton production has been discussed in the literature [3]. The CGHS model has its origin in the dimensional reduction of the more realistic, four dimensional, Einstein-Hilbert gravitation with a spherically symmetric metric. The dilaton field in the resulting model is a relic of the angular variables hidden in this procedure. Also, it belongs to the class of models first discussed by Jackiw and Teitelboim [4], which lend themselves to a gauge theoretical formulation [4, 5] of the problem.

The formation of black holes in bidimensional dilaton gravity coupled to scalar field was considered recently in different contexts. For instance, the scalar matter field was already considered in the form of sine-Gordon [6] and quartic potentials [7], and these investigations have motived us to introduce new systems for the scalar matter.

In this paper we investigate the formation of black hole in two different systems. The first system considers a sixth-order potential for the matter field that couples to the bidimensional dilaton gravity. This is a new possibility, and although we yet have polynomial potential, the nature of the solutions are different from the one that appear with the quartic potential already investigated, since in the sixth-order potential the kink connects the symmetric vacuum to an asymmetric one. The second system we shall deal with is a system of two coupled fields recently introduced [8]. This system presents very interesting properties [9, 10], and here we show that the second field adds further effects.

This work is organized as follows. In the next Sec. 2 we present the CGHS model with a single field system, given by a sixth-order potential, in Sec. 2.1. The two field system as the source of matter is worked out in Sec. 2.2 and again we have black hole solutions, now controlled by an enlarged number of parameters. Discussions and final remarks are introduced in Sec 3.

2 Models of Matter Fields

In this section we deal with two different examples of coulping dilaton gravity to matter fields. The first example represents a single field system that contains up to sixth power self-interaction terms. The second example is different since it contains two coupled scalar fields, and is defined by a potential that contains up to the quartic power in the fields, and presents an enlarged set of parameters.

2.1 System with a single field

Here we consider the model described by the action

$$\begin{aligned} S &= \frac{1}{2\pi}\int d^2x\sqrt{-\bar{g}}\Big\{ e^{-2\phi}\Big[\bar{R}+4(\bar{\nabla}\phi)^2+4\lambda^2\Big] \\ &\quad -\frac{1}{2}(\bar{\nabla}f)^2+2\mu^2 f^2(f^2-a^2)^2 e^{-2\phi}\Big\} \ , \end{aligned} \tag{1}$$

where $\bar{g}$, ϕ and f are the metric, dilaton and matter fields respectively. $\bar{R}$ is the scalar curvature and λ^2 is a cosmological constant. This action is the usual action, except that the last term contains a specific sixth-order potential for the self-interacting matter field.

We can be write Eq. (1) in a different form, using a rescaled metric tensor $g_{\mu\nu}$ in such a way that

$$\bar{g}_{\mu\nu} = e^{2\phi} g_{\mu\nu} \ . \tag{2}$$

In this case, the action given by Eq. (1) turns into

$$\begin{aligned} S &= \frac{1}{2\pi}\int d^2x\sqrt{-g}\Big[e^{-2\phi}R+4\lambda^2 \\ &\quad -\frac{1}{2}(\nabla f)^2+2\mu^2 f^2(f^2-a^2)^2\Big] \ . \end{aligned} \tag{3}$$

Note that this conformal reparametrization of the field eliminates the kinetic term for the dilaton that appears in the original action.

The equations of motion that follow from Eq. (3) are

$$\nabla^2(e^{-2\phi}) - 4\lambda^2 - 2\mu^2 f^2(f^2-a^2)^2 = 0 \ , \tag{4}$$

and

$$R = 0 \ . \tag{5}$$

Equation (5) implies that $g_{\mu\nu} = \eta_{\mu\nu}$ and using the fact that in two dimensions we can always put the metric in the conformal gauge

$$\bar{g}_{\mu\nu} = e^{2\rho}\eta_{\mu\nu} \ , \tag{6}$$

we conclude that $\rho = \phi$.

In order to investigate the solutions of this model it is useful to introduce light-cone coordinates $x^{\pm} = t \pm x$. In these coordinates the line element constructed with the metric tensor $g_{\mu\nu}$ is given by

$$ds^2 = -dx^+dx^- \ . \tag{7}$$

In terms of these coordinates, the action given by Eq. (3) can be cast to the form

$$S = \frac{1}{\pi}\int d^2x\Big[\Big(2e^{-2\phi}\partial_+\partial_-\rho+\lambda^2\Big) -\frac{1}{2}\partial_+ f\partial_- f-\frac{1}{2}\mu^2 f^2(f^2-a^2)^2\Big]. \tag{8}$$

The field equations of motion are given by

$$\partial_+^2\left(e^{-2\phi}\right)+\frac{1}{2}\left(\partial_+ f\right)^2 = 0, \tag{9}$$

$$\partial_-^2\left(e^{-2\phi}\right)+\frac{1}{2}\left(\partial_- f\right)^2 = 0, \tag{10}$$

$$\partial_+\partial_-\left(e^{-2\phi}\right)+\lambda^2-\frac{1}{2}\mu^2 f^2(f^2-a^2)^2 = 0, \tag{11}$$

$$\partial_+\partial_- f+\mu^2 f(f^2-a^2)(3f^2-a^2) = 0, \tag{12}$$

and these are the equations we have to deal with.These results follow very naturally in the above procedure, and this should be contrasted to the procedure used in Refs. [6, 7] where one has to appropriately choose the gauge to get to such r esult. Here we see that the equation of motion for the matter field presents solutions that can be cast to the form, working with standard coordinates (x,t)

$$f^2(x,t)=\frac{1}{2}a^2\{1+\tanh[\alpha((x-\bar{x})+v(t-\bar{t}))]\}, \tag{13}$$

where α is a constant and $(\bar{x},\bar{t})$ represents the center of the kink. In the light-cone coordinates we can write the above solutions as

$$f^2(x^+,x^-)=\frac{1}{2}a^2\{1+\tanh[\alpha_+(x^+-\bar{x}^+)-\alpha_-(x^--\bar{x}^-)]\}, \tag{14}$$

where $\alpha_\pm=\frac{\alpha}{2}\,(v\pm 1)$.

In order to study black hole formation we use this f into the other equations of motion for ϕ. Here we can write, for instance,

$$\partial_+\partial_-\left(e^{-2\phi}\right)=\frac{\mu^2a^6}{16}\Big[1+\tanh^3\triangle-\tanh^2\triangle-\tanh\triangle\Big]-\lambda^2, \tag{15}$$

where we have set $\triangle=\delta-\bar{\delta}$, with $\delta=\alpha_+x^+-\alpha_-x^-$ and $\bar{\delta}=\alpha_+\bar{x}^+-\alpha_-\bar{x}^-$. This equation can be integrated to give

$$e^{-2\phi}=C_1+b(x^+)+d(x^-)-\lambda^2x^+x^-+\frac{\mu^2a^6}{16\alpha_+\alpha_-}\Big[\frac{1}{2}\tanh\triangle-\ln\cosh\triangle\Big]. \tag{16}$$

The functions $b(x^+)$ and $d(x^-)$ are determined by the constraint equations, the two first equations of motion (8) and (9). Here we get

$$b(x^+) = b\,x^+ + C_2, \tag{17}$$
$$d(x^-) = d\,x^- + C_3. \tag{18}$$

Therefore, the dilaton field can be determined up to constants in the form

$$e^{-2\phi} = C + b\,x^+ + d\,x^- - \lambda^2 x^+ x^- + \frac{\mu^2 a^6}{16\alpha_+\alpha_-}\Big[\frac{1}{2}\tanh\triangle - \ln\cosh\triangle\Big], \tag{19}$$

where b, d, and C are constants, and in the following we choose $b = d = 0$, for simplicity.

Let us now investigate the geometric nature of this solution generated by original system. Toward this goal, let us divide spacetime into the three regions: $\triangle = \delta - \bar{\delta} << 1$, $\triangle \approx 0$, and $\triangle >> 1$. In the first region the dilaton field becomes

$$e^{-2\phi} \approx C - \lambda^2\left(x^+ + \frac{\mu^2 a^6}{16\lambda^2\alpha_+}\right)\left(x^- - \frac{\mu^2 a^6}{16\lambda^2\alpha_-}\right) \ , \tag{20}$$

where the constant C is taken as

$$C = \frac{\mu^2 a^6}{16\alpha_+\alpha_-}\left(\bar{\delta} + \frac{\mu^2 a^6}{16\lambda^2} - \ln 2 - \frac{1}{2}\right). \tag{21}$$

In the region where $\triangle = \delta - \bar{\delta} \approx 0$ we get

$$e^{-2\phi} \approx \frac{\mu^2 a^6}{16\alpha_+\alpha_-}\left(\bar{\delta} + \frac{\mu^2 a^6}{16\lambda^2} - \ln 2 - \frac{1}{2}\right) - \lambda^2 x^+ x^-. \tag{22}$$

With regard to the initial metric $\bar{g}_{\mu\nu}$, these solutions represent, where this metric is defined [1], the linear dilaton vacuum.

In the third region we have $\triangle = \delta - \bar{\delta} >> 1$, and now we obtain

$$e^{-2\phi} \approx \frac{\mu^2 a^6}{16\alpha_+\alpha_-}(2\bar{\delta}) - \lambda^2\left(x^+ - \frac{\mu^2 a^6}{16\lambda^2\alpha_+}\right)\left(x^- + \frac{\mu^2 a^6}{16\lambda^2\alpha_-}\right), \tag{23}$$

which represents, with respect to the original background,the geometry of a black hole with mass

$$\frac{\lambda\mu^2 a^6}{8\alpha_+\alpha_-}\bar{\delta}, \tag{24}$$

after shifting x^+ by $\mu^2 a^6/16\lambda^2\alpha_+$ and x^- by $-\mu^2 a^6/16\lambda^2\alpha_-$.

2.2 System with two fields

Let us now consider another system, defined by

$$
\begin{aligned}
S &= \frac{1}{2\pi}\int d^2x\sqrt{-g}\Big\{\left[e^{-2\phi}R + +4\lambda^2\right] \\
&\quad -\frac{1}{2}(\nabla f)^2 - \frac{1}{2}(\nabla g)^2 + U(f,g),\Big\}\ . \qquad (25)
\end{aligned}
$$

This action can be written in terms of $\bar{g}_{\mu\nu}$ by doing the inverse transformation that correspondes to Eq. (2).

Using light-cone coordinates the above action can be cast to the form

$$
\begin{aligned}
S &= \frac{1}{\pi}\int d^2x\Big[\left(2e^{-2\phi}\partial_+\partial_-\rho + \lambda^2\right) \\
&\quad +\frac{1}{2}\partial_+f\partial_-f + \frac{1}{2}\partial_+g\partial_-g - \frac{1}{4}U(f,g)\Big]. \qquad (26)
\end{aligned}
$$

This new system is defined via the potential

$$
U(f,g) = \frac{1}{2}\mu^2(f^2-a^2)^2 + \mu\nu(f^2-a^2)g^2 + \frac{1}{2}\nu^2g^4 + 2\nu^2f^2g^2\ . \qquad (27)
$$

This potential identifies a system of two real scalar fields that was recently investigated in [8], where it was shown to present interesting static field configurations. As we can see from the above potential, we are now dealing with a richer system and we want to explore how this enlarged system change the simpler picture of black hole formation in systems of a single field.

We follow as in the former system.In this case we get

$$
\begin{aligned}
\partial_+^2(e^{-2\phi}) + \frac{1}{2}(\partial_+f)^2 + \frac{1}{2}(\partial_+g)^2 &= 0\ , \qquad (28) \\
\partial_-^2(e^{-2\phi}) + \frac{1}{2}(\partial_-f)^2 + \frac{1}{2}(\partial_-g)^2 &= 0\ , \qquad (29) \\
\partial_+\partial_-(e^{-2\phi}) + \lambda^2 - \frac{1}{4}U(f,g) &= 0\ , \qquad (30) \\
\partial_+\partial_-f + \frac{1}{2}\mu^2(f^2-a^2)f + \nu(\nu + \frac{1}{2}\mu\, fg^2 &= 0\ , \qquad (31) \\
\partial_+\partial_-g + \frac{1}{2}\mu\nu(f^2-a^2)g + \frac{1}{2}\nu^2g^3 + \nu^2f^2g &= 0\ . \qquad (32)
\end{aligned}
$$

The above Eqs. (31) and (32) correspond to the matter field equations of motion. They can be solved to give two different pair of solutions [8]:

$$
f = -a\tanh\{\mu a[\alpha_+(x^+ - \bar{x}^-) - \alpha_-(x^- - \bar{x}^-)]\}, \qquad (33)
$$

and $g = 0$, and also

$$f = -a \tanh\{2\nu a[\alpha_+(x^+ - \bar{x}^+) - \alpha_-(x^- - \bar{x}^-)]\} \; , \tag{34}$$

and

$$g = \pm a \left(\frac{\mu}{\nu} - 2 \right)^{1/2} \operatorname{sech}\{2\nu a[\alpha_+(x^+ - \bar{x}^+) - \alpha_-(x^- - \bar{x}^-)]\} \; , \tag{35}$$

valid for $\mu/\nu > 2$. Here we note that the limit $\nu \to \mu/2$ transforms the second pair of solutions into the first one, that presents $g = 0$. This is interesting since the investigation of the second pair of solutions allows getting results valid for the first pair, with presents $g = 0$ and so leads to the case of just one field. See below for further details.

For the second pair of solutions we can cast the dilaton field in the form

$$e^{-2\phi} = C - \lambda^2 x^+ x^- - A \tanh^2[2\nu a(\delta - \bar{\delta})] - B \ln \cosh[2\nu a(\delta - \bar{\delta})] \; . \tag{36}$$

Like in the former case, in the above expression we have also chosen $b = d = 0$ in $b(x^+) = bx^+ + C_2$ and $d(x^+) = dx^+ + C_3$, which follow from (28) and (29). The expressions for A and B are given by

$$A = \frac{a^2}{12\nu^2 \alpha_+ \alpha_-} \left[\frac{\mu^2}{4} + \frac{1}{4}(\mu - 2\nu)(7\mu + 10\nu) \right] \; , \tag{37}$$

and

$$B = \frac{a^2}{12\nu^2 \alpha_+ \alpha_-} \left[\mu^2 + (\mu - 2\nu)(\mu - 4\nu) \right] \; . \tag{38}$$

Let us now investigate the geometric nature of the black hole generated in this system. Here we can write the dilaton field ,for $\Delta >> 1$, as

$$e^{-2\phi} \approx 4aB\nu\bar{\delta} - \lambda^2 \left(x^+ - 2\frac{\nu}{\lambda^2} Aa\alpha_- \right) \left(x^- + 2\frac{\nu}{\lambda^2} Aa\alpha_+ \right) \; , \tag{39}$$

which represents the geometry of a black hole with regard to the original background spacetime.The mass of the black hole is given by

$$4aB\lambda\nu \left(\bar{\delta} + \frac{\nu}{\lambda^2} Aa\alpha_+\alpha_- \right) \; . \tag{40}$$

after shifting x^+ by $2(\nu/\lambda^2)aB\alpha_-)$ and x^- by $-2(\nu/\lambda^2)aB\alpha_+$.

For $\Delta = \bar{\delta} - \delta << 1$,

$$e^{-2\phi} \approx -\lambda^2 \left(x^+ + 2\frac{\nu}{\lambda^2} Aa\alpha_- \right) \left(x^- - 2\frac{\nu}{\lambda^2} Aa\alpha_+ \right) \; , \tag{41}$$

where the constant C was taken as

$$C = A + 2aB\nu\bar{\delta} - B \ln 2 - \frac{4\nu^2 a^2 B^2 \alpha_+ \alpha_-}{\lambda^2} \; . \tag{42}$$

In the region of $\Delta \approx 0$ we have

$$e^{-2\phi} \approx A - B\ln 2 + 2aB\nu\bar{\delta} - \frac{4a^2B^2\nu^2\alpha_+\alpha_-}{\lambda^2} - \lambda^2 x^+ x^- \ , \tag{43}$$

As in the first system, these last two solutions give us the linear dilaton vacuum in the region where the original background is defined.

We recall that the limit $\nu \to \mu/2$ changes the second pair of solutions that we have been considering to the first one, simpler, that presents $g = 0$, as introduced above. For this reason, we can investigate this first pair of solutions by just setting $\nu \to \mu/2$ in the above results.

This procedure is interesting since it leads to the case with just one field, more precisely to the case where the matter system is described by the ϕ^4 model, but this was already investigated in Ref. [7]. Despite slightly different notations, it is not hard to see that the limit $\nu \to \mu/2$ correctly change the above results into the results given in [7] for the case of a single field, in the ϕ^4 model for the matter contents.

3 Comments and Conclusions

In this work we have investigated bidimensional dilaton gravity coupled to two different matter field systems. The first system is a single field system that contains self-interactions up to the sixth power. It is of the same kind of the sine-Gordon [6] and $\lambda\phi^4$ [7] models already investigated. The results of these papers, together with the present work show that the black hole that appears is qualitatively the same, but with different masses that depend on the parameters associated with the various solutions. The second system is a system of two fields, and contains up to the fourth power in the fields. This system is richer since it is defined in a enlarged space of parameters, which contains the space of parameters of the $\lambda\phi^4$ system as a particular case. The soliton solutions that appear in this case also contributes to the generation of black holes, but these black holes are quantitatively different from black holes that appear in systems of a single field, the difference being controlled by the enlarged set of parameters that defines the two field matter system.

Evidently, one can have more examples, for instance investigating the case where the scalar matter is described by the two field system that contains up to the sixth power in the fields, as also investigated in [8]. We think that it is interesting to investigate this kind of mechanism for different matter field potentials, in order to understand mechanisms of formation of black holes within the context of dilaton gravity. Furthermore, there is also the newer context of the Einstein-Maxwell-dilaton-axion system [11] with general dilaton coupling. And this naturally leads to another interesting issue, that concerns investigating the coupling of a dilaton-axion gravitational field with two coupled scalar fields, in the form introduced in [8, 9]. The two cases worked out above lead us to different scenario

concerning formation of black holes, and so it turns out to be interesting to analyze quantum or at least semiclassical versions [5] since these black holes may present new termodinamics properties, namely the Hawking-Bekenstein radiation. Work on this and in other related issues is now in progress.

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